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The impacts of outliers on different estimators for GARCH processes: an empirical study

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The impacts of outliers on different estimators for GARCH processes: An empirical study

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Abstract

The Maximum likelihood estimation (MLE) is the most widely used method to estimate the parameters of a GARCH(p,q) process. This is owed to the fact that the MLE, among other properties, is asymptotically efficient. Even though the MLE is sensitive to outliers, which can occur in time series. In order to abate the influence of outliers, robust estimators are introduced. Afterwards an Monte Carlo study compares the introduced estimators.
Keywords: GARCH, Robust-Estimates, M-Estimates

1 Introduction

The ARCH model introduced by Engle [1982] and extended to the GARCH model by Bollerslev [1986] is able to capture some of the stylized facts of financial data Rama [2001]. The analysis of financial time series is sensitive to outliers, due to the temporal dependence in the data. According to Carnero et al. [2007] the conditional heteroscedasticity of financial data could be explained through the presence of outliers in the data. Outliers can be explained, among other things, via errors in the transmission or deviations from the assumed model. This motivates the use for robust estimators for GARCH models. Section 2 summarizes current results for different estimators for GARCH processes. In section 3 several robust estimators for a GARCH(p,q) process are introduced. A comparative Monte Carlo study with different types of outliers and estimators is presented in section 4 and the appendix. This simulative study extends the study of Muler and Yohai [2008] adding more type of outliers and introducing non-equidistant outliers. In section 5 four financial time series are analyzed with the estimators presented in chapter 3.

2 Estimators for GARCH processes

In this paper only estimators for centered GARCH(p,q) processes are considered. This is no limitation, since the underlying process can be decomposed in a trend component and a centered GARCH(p,q) process.

Definition: *Centered GARCH(p,q) process*

A stochastic process x_t is said to be a centered GARCH(p,q) process, if:

$$x_t|\mathcal{F}_{t-1} \sim N(0, \sigma_t),$$
$$\sigma_t^2 = (\sigma_t(\gamma))^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i x_{t-i}^2 + \sum_{i=1}^{q} \beta_i \sigma_{t-i}^2, \quad t \in \mathbb{Z}$$

with $\gamma = (\alpha_0, \alpha_1, \cdots, \alpha_q, \beta_1, \cdots, \beta_q)$, $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1, \ldots, p$ *and* $\beta_i \geq 0$, $i = 1, \ldots, q$,

where $\mathcal{F}_t$ denotes the information set of the process up to time t. The process is weak stationary if $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_j < 1$ (Bollerslev [1986]). If $A(x) = \sum_{i=1}^{p} \alpha_i x^i$ and $B(x) = 1 - \sum_{i=1}^{q} \beta_i x^i$ are coprimes, the parameters of the process are non-ambiguous Berkes et al. [2003]
From this definition there are three different representations of the process x_t, if the process is weak stationary, leading to different feasible methods to estimate the parameters of the process. The explicit representation of the conditional variance is:

$$\sigma_t^2 = \frac{\alpha_0}{1 - \sum_{i=1}^{p} \beta_i} + \sum_{i=1}^{p} \alpha_i x_{t-1}^2 + \sum_{i=1}^{p} \alpha_i \sum_{k=1}^{\infty} \sum_{j_i=1}^{q} \cdots \sum_{j_k=1}^{q} \beta_{j_1} \ldots \beta_{j_k} x_{t-i-j_1-\ldots j_k}^2. \tag{2.1}$$

Since our observations start at time $t = 1$, one can set $x_t = 0$ for $t \leq 0$. For the asymptotic results there is no difference whether the conditional variance σ_t defined by the whole process or the conditional variance $\widetilde{\sigma}_t$, defined by the truncated process, is used for estimating the parameter Muler and Yohai [2008].

$$\widetilde{\sigma}_t = \frac{\alpha_0}{1 - \sum_{i=1}^{q} \beta_i} + \sum_{i=1}^{p} \alpha_i x_{t-1}^2 + \sum^{p} \alpha_i \sum_{k=1}^{\infty} \sum_{j_i=1}^{q} \cdots \sum_{j_k=1}^{q} \beta_{j_1} \ldots \beta_{j_k} x_{t-i-j_1-\ldots j_k}^2 I_{t-i-j_1-\ldots j_k \geq 1}, \tag{2.2}$$

where I_A denotes the indicator function of A. The parameters of the process can be estimated via the (quasi) maximum likelihood method. Every weak stationary GARCH(p,q)-Process $(x_t)_{t \in T}$ has an ARMA$(\max(p,q), q)$-representation of the squared observations.

$$x_t^2 = \alpha_0 + \sum_{i=1}^{max(p,q)} (\phi_i) \cdot x_{t-i}^2 + \sum_{i=1}^{q} \theta_i \cdot v_{t-j} + v_t.$$

The process $v_t = x_t^2 - \sigma_t^2 =$ is a martingale difference sequence in respect to the filtration $\mathcal{F}_{t-1}$, since:

$$E[v_t|\mathcal{F}_{t-1}] = E[x_t^2 - \sigma_t^2|\mathcal{F}_{t-1}] = E[(z_t^2 - 1)\sigma_t|\mathcal{F}_{t-1}] = E[z_t^2 - 1](\sigma_{t-1}) = 0.$$

The parameters of the process can be estimated by solving the resulting Yule-Walker equations.
Every weak stationary GARCH(p,q)-Process has an ARCH(∞)-representation:

$$\sigma_t = \alpha_0^\star + \sum_{i=0}^{\infty} \alpha_i^\star \cdot x_{t-i}^2$$

The parameters can be obtained by an OLS or MLE. Under certain regularity conditions all estimators mentioned in this section are asymptotically normal distributed. The exact conditions for the MLE can be found in Bollerslev [1986] or Francq and Zakoïan [2004], for the OLS in Engle [1982] and for the estimator based on the Yules-Walker equations in Kristensen and Linton [2006] or Franke et al. [2004].
In the next section some results for the robust properties for these estimators are presented.

3 Robust estimators for GARCH processes

The robust features of an estimator can be quantified through qualitative robustness, the influence function and the breakdown point (Huber [1981]). According to Huber a robust estimator should be qualitative robust, have a bounded influence function and a breakdown point, which is strictly larger than zero. None of the above mentioned estimators have any of these robust characteristic. The QMLE and the OLS have an unbounded influence function and a breakdown point of zero Mendes [2000] and Li [1985]. The estimator based on solving the Yule-Walker equations is not robust either, since it is based on the autocorrelation function which is not robust Ma and Genton [2000]. The QML-estimator is a special case of a much broader class of estimators, namely the class of M-estimators.

Definition: *M-estimator*

Every estimator $\theta_n \subset \theta$, $\theta \in \mathbb{R}^p$ defined explicit through the minimization of

$$\min_{\theta_n \in \theta} \sum_{i=1}^{n} \rho(x_i, \theta_n)$$

is an M-estimator. Where $\rho : \mathbb{R} \times \mathbb{R}^p \to \mathbb{R}$ is an arbitrary function. If ρ has a continuous derivative $\psi(x, \theta) = \frac{\partial \rho(x,\theta)}{\partial \theta}$, the estimate

$$\sum_{i=1}^{n} \psi(x_i, \theta_n) = 0$$

is said to be defined implicit.

As Huber [1981] shows, the influence function depends on the function ρ. Denote with $z_t = \frac{x_t}{\sigma_t}$ the innovations of a GARCH(p,q) process. The MLE can be expressed as an M-estimator :

$$\hat{\gamma}_{MLE} = \arg\min_{c \in C} \frac{1}{T-p} \sum_{t=p+q}^{T} \rho_{ML}(\ln z_t^2),$$

where $\rho_{ML} = -\ln g_0, g_0$ is the density of $\log z_t^2$ $z_t \sim N(0,1)$ and $c \in \mathcal{C}$, $\mathcal{C}$ a compact subspace of $\mathbb{R}^{1+p+q}$ i.e.

$$C_\delta := \left\{ a \in \mathbb{R}_+^{p+1}, b \in \mathbb{R}_+^q \;\middle|\; a_0 \in [\delta, 1/\delta], \sum_{i=1}^{p} a_i \geq \delta, \sum_{i=1}^{p} a_i + \sum_{i=1}^{q} b_i \leq 1 - \delta \right\}$$

A straight forward attempt to obtain a robust estimator would be to constrain ψ, the derivative of ρ, trading off efficiency for robustness. The LAD-estimator introduced by Peng and Yao [2003] with $\rho(u) = |u|$ and $\psi(u) = sign(u)$ and the MLE with t-distributed innovations belong to this class. To gain even more robustness from outliers, one could constrain ρ, too. Among this class are estimators, of the form $\rho = m(\rho_{ML})$ where m is a bounded function. Muler and Yohai [2008] introduce these as m-estimators. Under the following conditions both classes of estimators are asymptotically normal distributed:

Theorem

Let the GARCH(p,q) process be stationary, the parameters are unique, the density g of the log squared innovations is unimodal continuous and positive. Moreover let $\rho = m(-\ln(g))$ and m is a monotone function and the following conditions hold:

(i) ρ *has a bounded derivative,*

(ii) *the true parameter* $\gamma \in C_\delta$,

(iii) ρ *has three continuous and bounded derivatives,*

(iv) $E(\psi^2(w_t)) > 0$ *and*

(v) $E(\psi'(w_t)) > 0$.

Then the M-estimator is asymptotically normal distributed:

$$T^{\frac{1}{2}}(\hat{\gamma}_N - \gamma) \xrightarrow{\mathcal{D}} N(0, V),$$

$$\text{where } V = \frac{E_g(\psi^2(w_t))}{E_g^2(\psi'(w_t))} \left(E_g \left(\frac{1}{\sigma_t^2(\gamma)} \nabla \sigma_t^2(\gamma) (\nabla h_t^2(\gamma))^t \right)^{-1} \right),$$

where $\nabla \sigma_t^2$ *denotes the gradient of* (σ_t^2) *and* $(\sigma_t^2)^t$ *denotes the transposed of* (σ_t^2).

The condition on the density g of the log squared innovations is not necessary for the asymptotic behavior. The estimated parameters $(a_0, a_1, \ldots, a_p)$ need to be corrected by the

factor e^{-u_0} for consistency, where u_0 is the minimum of $J(u) = E(\rho(\log z_t^2 - u))$ The proof can be found in Muler and Yohai [2008].
Muler and Yohai [2008] propose a slightly different approach by modifying the conditional variance slightly, thus gaining more robustness. They propose "bounded M-Estimators"(BM-estimators). To reduce the influence of an outlier on conditional variance, define:

$$(\sigma_{t,k}^\star)^2 = (\sigma_{t,k}^\star(c))^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i (\sigma_{t-i}^\star(c))^2 r_k \left(\frac{x_{t-i}^2}{(\sigma_{t-i}^\star(c))^2} \right) + \sum_{i=1}^{q} \beta_i (\sigma_{t-i}^\star(c))^2, \tag{3.1}$$

where $c = (a_0, a_1, \ldots, a_p, b_1, \ldots, b_q)$ is the parameter vector and $x_t = 0$ for $t \leq 0$ and

$$r_k(u) = \begin{cases} u & \text{if} \quad u \leq k \\ k & \text{if} \quad u > k. \end{cases}$$

If k is chosen large enough σ_t^2 and $(\sigma_{t,k}^\star)^2$ are corresponding.
Under the assumption of the conditional normal-distribution $z_t^2 = \frac{x_t^2}{\sigma_t^2} \sim \chi^2(1)\ \forall t \geq 2$. So k can be chosen as quantile of a χ^2-distribution with one degree of freedom.
The BM-estimator is defined as:

$$\hat{\gamma}_2 = \arg\min_{c \in \mathcal{C}} (M_{t,k}(c)), \tag{3.2}$$

where

$$M_{Tk}^\star(c) = \frac{1}{T-p} \sum_{t=p+1}^{T} \rho(2\ln(x_t) - 2\ln(\sigma_{t,k}^\star(c))^2).$$

To increase the efficiency of the estimator the BM-estimator can be augmented to:

$$\gamma^B = \begin{cases} \hat{\gamma}_1 & \text{if} \quad \widetilde{M}_T(\hat{\gamma}_1) \leq M_{Tk}^\star(\hat{\gamma}_2) \\ \hat{\gamma}_2 & \text{if} \quad \widetilde{M}_T(\hat{\gamma}_1) > M_{Tk}^\star(\hat{\gamma}_2), \end{cases} \tag{3.3}$$

where $\hat{\gamma}_1 = \arg\min_{c \in C_\delta} \widetilde{M}_T(\hat{\gamma}_1)$ is the MLE and $\hat{\gamma}_2 = \arg\min_{c \in C_\delta} M_{Tk}^\star(c)$.

Theorem

If the distribution of the innovations gives positive probability to the complement of any compact and $\lim_{|u| \to \infty} \rho(u) = \sup_u \rho(u)$ and the above mentioned conditions hold, the same asymptotic result as for M-estimators holds for the BM-estimators.

The proof can be found in Muler and Yohai [2008].

4 Simulation

4.1 Simulation design

The following monte carlo study compares different estimators for GARCH parameters in the presence of outliers. A GARCH(1,1) process with the parameters $\phi = (\alpha_0,\ \alpha_1,\ \beta_1) = (0.1, 0.19, 0.8)$ with 1000 observations and a burn-in period of 200 is simulated and contamined with different types of outliers. The parameters are chosen to be similar to the Nikkei 225 Index Mittnik et al. [1998] since the behavior of this series is rather typical for financial return data. In their study Mittnik. et al. found, that a GARCH(1,1) process with the above mentioned parameters is adequate for the squared returns of the Nikkei 225 Index. To compare the different estimators, the mean square error (MSE) is used as a measure for the goodness of fit.

$$MSE = E\big((\hat{\theta} - \theta)^2\big)$$

The following estimates are compared:

Estimate	$\rho(u)$
QML	$-\ln(g(u))$
t-GARCH	$-\ln\left(\frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\Gamma(\frac{n}{2})}(1+\frac{e^u}{n})^{\frac{-n+1}{2}}e^u\right)$
SML	$2\ln(1+e^u) - u/2$
LAD	$\lvert u \rvert$
M1	$m_1(-\ln(g(u)))$
M2	$0.8m_1(-\ln(g(u))/0.8)$
BM1	$m_1(-\ln(g(u)))$
BM2	$0.8m_1(-\ln(g(u))/0.8)$

Where $\rho(u)$ is the function to be minimized for the M-estimator. BM1 and BM2 are BM-estimates, defined in equation 3.3, where $k = 5.02$ respectively $k = 2.72$. For the t-GARCH the degrees of freedom are also estimated. To save computational time, an upper bound for the degrees of freedom is set at 150, since the difference for the density of a normal distribution and a t-distribution with degrees of freedom=150 is smaller than 0.001. The function g is the density of the log squared innovations $w = \ln(z_t^2)$. If the innovations are normal distributed $z_t \sim N(0,1)$, the density can be written as:

$$g(w) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(e^w - w)}$$

and the function m_1 is defined by:

$$m_1(x) = \begin{cases} x & \text{if} \quad x \leq 4 \\ P(x) & \text{if} \quad 4 < x \leq 4,3 \\ 4,15 & \text{if} \quad x > 4.3, \end{cases}$$

where $P(x)$ is a spline of degree 4 with the following constraints:
$P(a) = a$, $P'(a) = 1$, $P'(b) = P''(a) = P''(b) = 0$. The spline is used to guarantee that $m_1(x)$ is continuous. The function m_1 and the side constraints for the spline are taken from Muler and Yohai [2008].

4.2 Outliers

The outliers considered in the study are introduced by Fox [1972] as type I and type II outliers. The contaminated time series $x_t^\star$ is derived from a simulated GARCH(1,1)-process x_t. Type I outliers are "gross-error of observations or recording error that affects a single observation" Fox [1972] and can be modeled as:

$$x_t^\star = x_t + d\sigma_t u_t,$$

where $d \in \mathbb{R}$, σ_t the conditional variance and $u_t \overset{\text{i.i.d.}}{\sim} \text{Ber}(p)$. The clustered outliers are modeled similar to the additive outliers with the difference that if at time t_i an outlier occurs, the next m observations are also modeled as additive outliers. The length m is random and modeled with a Poisson distribution with varying $\lambda = [3,\ 5,\ 7]$. The size of the outliers are fixed with $d = 5$. Type II outliers correspond to a different innovation, in this simulation the innovation $z_t^\star$ is not normal but:

$$z_t^\star = \epsilon N(0,1) + (1-\epsilon)t_n,$$

where n is the number of degrees of freedom.

4.2.1 Scenarios

The following scenarios are conducted:

additive outliers	p	0 %	5 %	10 %
	size	3	5	7
clustered additive outliers	p	0 %	5 %	10 %
	λ	3	5	7
outliers in the innovations	ϵ	0 %	5 %	10 %
	degrees of freedom	3	4	5

Table 1: Scenarios for the Monte Carlo study.

4.3 Results

4.3.1 Additive outliers

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	0,5768	0,1363	0,8095	0,5014	0,0082	0,0109
t-Garch [1]	0,2762	0,1178	0,8051	0,0936	0,0086	0,0089
LAD	0,2967	0,1072	0,7831	0,1441	0,0127	0,0221
SML	0,2875	0,1343	0,7379	0,0528	0,0033	0,0049
M1	0,2585	0,1632	0,7551	0,0770	0,0055	0,0123
M2	0,2988	0,1816	0,7449	0,1018	0,0074	0,0160
BM1	0,1145	0,1958	0,7844	0,0035	0,0020	0,0021
BM2	0,2873	0,2917	0,7074	0,0408	0,0120	0,0102

Table 2: Estimates and MSE for a GARCH(1,1) model with additive outliers, $p = 5\%$ and outlier size $d = 3$.

Table 2 shows the estimated parameters and the MSE for a GARCH(1,1) process with $p = 5\%$ and outlier size $d = 3$. All remaining scenarios are shown in appendix A. All but the BM1-estimator overestimate the parameter α_0. With increasing p and d the overestimation gets worse. There seems to be a negative correlation of the estimate for β_1 and outlier size d for all but the BM1 and BM2 estimator. The t-GARCH estimator has one parameter more to estimate than SML-estimator and has slightly worse estimates. The best results are achieved with the BM1-estimator though it seems that with increasing contamination and increasing size of the outliers that the BM2-estimator outperforms the BM1-estimator in terms of a lower MSE. The Monte Carlo simulation of Muler and Yohai [2008] reinforces this impression, even though their simulation is carried out with equidistant outliers.

4.3.2 Clustered additive outliers

Table 3 shows the estimated parameters and the MSE for a GARCH(1,1) process with clustered additive outliers, $p = 5\%$, cluster-length $\lambda = 3$ and outlier size $d = 5$. All estimators overestimate α_1 and underestimate β_1. The LAD estimator even indicates, that the process is non stationary. With increasing cluster length the miss-estimation for the parameter α_0 for the QML and the LAD estimator decreases, while for the other estimators the contrary holds. The other parameters stay constant for the QML and LAD estimator, while for the other estimators the estimate for α_1 decreases slightly and increases slightly for β_1. This

[1] Estimated degrees of freedom $=3,7804$

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	2,4161	0,2905	0,5975	13,1240	0,0303	0,1058
t-Garch [2]	1,7651	0,7726	0,1756	3,6612	0,3602	0,4118
LAD	1,7286	0,9838	0,0584	3,0264	0,6503	0,5543
SML	0,4327	0,1458	0,7204	0,2385	0,0035	0,0120
M1	1,2790	0,6385	0,2131	1,6819	0,2066	0,3592
M2	1,1704	0,6676	0,2493	1,4716	0,2379	0,3273
BM1	0,5744	0,5395	0,4569	0,3626	0,1399	0,1369
BM2	0,4500	0,3759	0,6235	0,1943	0,0466	0,0433

Table 3: Estimates and MSE for a GARCH(1,1) model with clustered additive outliers, $p = 5\%$, cluster size $\lambda = 3$ and outlier size $d = 5$.

could be explained by the fact, that shorter cluster resemble high peaks, which leads to high estimate for α_0 with the QML estimator. With increasing p, the t-GARCH estimator is the only estimator that can cope with this fact, but generally overestimates α_1. Larger cluster in the contaminated time series resemble the original GARCH process shifted up some units. Therefore the larger the cluster, the better are the QML and LAD estimators. The SML-estimator shows the smallest MSE for α_1 and β_1, while the BM1 estimator has the overall best result. With increasing λ the estimate for the parameter β gets closer to the value of the underlying process.

4.3.3 Outliers in the innovations

Tables 4 show the MSE for a GARCH(1,1)-process with contaminated innovations, $p = 5\%$ and $n = 5$. Not surprisingly the best performance is achieved with the classic QML-estimator. Since the fourth moment of the innovations does exists, the estimator is efficient Bollerslev and Wooldridge [1992]. If the fourth moment of the innovation does not exist, this estimator is not feasible. With increasing degrees of freedom the estimators become better, while with increasing p they get slightly worse. The t-GARCH estimator shows the best overall results, followed by the M1- and the LAD- estimator. In contrast the SML estimator, which is derived from a MLE with innovations that follow a t-distribution with 3 degrees of freedom, overestimates α_0 as well as the more robust estimators BM1 and BM2. It seems that the outliers in the innovations are 'mild', so that 'too much' robustness is born by efficiency.

[2] Estimated degrees of freedom $= 3,5231$

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	0,1373	0,2026	0,7869	0,0036	0,0011	0,0010
t-Garch [3]	0,1251	0,1969	0,7918	0,0031	0,0009	0,0008
LAD	0,1232	0,1892	0,7942	0,0059	0,0023	0,0022
SML	0,2498	0,1399	0,7319	0,0282	0,0025	0,0048
M1	0,1347	0,2038	0,7828	0,0036	0,0018	0,0016
M2	0,2251	0,2454	0,7506	0,0230	0,0053	0,0043
BM1	0,1623	0,2349	0,7588	0,0070	0,0037	0,0031
BM2	0,3446	0,2825	0,7173	0,0656	0,0097	0,0079

Table 4: Estimates and MSE for a GARCH(1,1) model with outliers in the innovations, $p = 5\%$ and $n = 5$.

5 Empirical results

Four financial time series with daily data are considered. First of all, serveral standard tests are carried out to check, whether there are GARCH effects in the data. If so a GARCH(1,1) process is fitted with the estimates described in the previous chapter. The adjusted daily stock price of the following companies is used.

Company	ISIN	Start Date	End Date
Agilent Technologies Inc.	US00846U1016	18.11.99	03.01.08
Integralis AG	DE0005155030	27.10.98	04.12.07
Bobst Group AG	CH0012684657	01.01.98	15.11.07
Cerespo Co., Ltd.	JP3423600000	01.01.98	15.11.07

Table 5: Names, ISIN and length of the used time-series.

A series of tests is conducted, namely the Kolmogorov-Smironv test, the Jarque-Bera test, the Lagrange multiplier and the robust Lagrange multiplier test. The robust Lagrange multiplier test is proposed in Bollerslev and Wooldridge [1992]. All tests, but the RLM-test for the Bobst Group AG, are significant on the confidence level of $\alpha = 0.05$, Table 22 summarizes the p-value of the tests. The different values for the LM-test and the RLM-test indicate the presence of outliers. According to the results of the tests a GARCH process is adequate to describe the time series.

[3] Estimated degrees of freedom = 150

The different estimator return different values for the model parameters (see table 23). Since the MSE is not applicable in this context, one way to evaluate the goodness of fit of the model is through the estimated innovations $\hat{z}_t$.

$$\hat{z}_t = \frac{x_t}{\hat{h}_t}.$$

For a M-estimate $\hat{\sigma_t}^2$ is

$$\hat{\sigma_t}^2 = \hat{\alpha_0} + \sum_{i=1}^{p} \hat{\alpha}_i x_{t-i}^2 + \sum_{i=1}^{q} \hat{\beta}_i \hat{\sigma}_{t-i}^2,$$

and for the BM-estimate $\hat{\sigma_t}^2$ is

$$\hat{\sigma_t}^2 = \hat{\alpha_0} + \sum_{i=1}^{p} \hat{\alpha}_i \hat{h}_{t-i} r_k \left(\frac{x_{t-i}}{\hat{\sigma}_{t-i}} \right) + \sum_{i=1}^{q} \hat{\beta}_i \hat{\sigma}_{tt-i}.$$

If the time series x_t follow a GARCH model, than it should hold for the estimated innovations

$$\sum_{t=1}^{T} (\hat{z}_t) = 0, \ \sum_{t=1}^{T} (\hat{z}_t^2) = 1.$$

Moreover they should be uncorrelated Muler and Yohai [2008]. Table 24 summarizes mean, standard deviation, Kendall τ and Spearman's ρ. The estimated mean and the rank correlations are close to zero for all estimates. In two cases the M1 estomator has the closest value to one, for the other two time series the same holds true for the M2 estimator. They behave slightly better than the two BM-estimates. This could be due to the fact, that the contamination is very mild.

6 Summary

Outliers have an effect on the estimation of the parameters for the assumed model. In the Monte Carlo simulation the BM-estimates had the best overall performance in the presence of outliers. Except in the case of outliers in the innovation. In the analysis of empirical data the empirical innovations of robust estimates have a better fit to the assumed model. The best performance is achieved with the M1- and the M2-estimators. They outperform slightly the BM1, respectively the BM2 estimators. The BM-estimator show a robust behavior, in the sense, that they are not influenced very much by outliers. The rigorous proof of the robust characteristic of the BM-estimator is yet to follow.

A Appendix

Results from the Monte-Carlo study

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	0,1228	0,1946	0,7893	0,0020	0,0010	0,0009
t-Garch [4]	0,1199	0,1915	0,7921	0,0024	0,0008	0,0009
LAD	0,0557	0,0885	0,7897	0,0031	0,0109	0,0027
SML	0,4848	0,2654	0,7345	0,1614	0,0058	0,0044
M1	0,1303	0,2004	0,7827	0,0034	0,0021	0,0018
M2	0,2073	0,2442	0,7501	0,0168	0,0055	0,0046
BM1	0,1228	0,1946	0,7893	0,0020	0,0010	0,0009
BM2	0,1228	0,1946	0,7893	0,0020	0,0010	0,0009

Table 6: Estimates and MSE for a GARCH(1,1) model with additive outliers, $p = 0\%$ and outlier size $d = 0$.

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	2,0853	0,2153	0,6484	12,7775	0,0035	0,0842
t-Garch [5]	1,0463	0,1136	0,6642	3,0212	0,0227	0,1190
LAD	0,7901	0,0823	0,6760	1,2082	0,0239	0,0925
SML	0,7001	0,1156	0,6873	1,2403	0,0086	0,0517
M1	0,6627	0,1971	0,6035	0,6030	0,0182	0,0867
M2	0,7462	0,2112	0,6024	0,8474	0,0289	0,1035
BM1	0,1098	0,1829	0,7852	0,0039	0,0017	0,0022
BM2	0,2595	0,2858	0,7117	0,0318	0,0116	0,0100

Table 7: Estimates and MSE for a GARCH(1,1) model with additive outliers, $p = 5\%$ and outlier size $d = 5$.

[4] Estimated degrees of freedom $=150$

[5] Estimated degrees of freedom $=2,5907$

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	2,6799	0,2260	0,6516	23,4838	0,0028	0,0832
t-Garch [6]	1,8313	0,1103	0,5793	8,1219	0,0416	0,2030
LAD	1,4039	0,0621	0,5468	3,4043	0,0335	0,1879
SML	1,1893	0,0859	0,6424	3,2253	0,0203	0,1219
M1	1,1365	0,1946	0,4885	1,9858	0,0344	0,2070
M2	1,2163	0,2012	0,5028	2,3308	0,0508	0,2109
BM1	0,1157	0,1718	0,7872	0,0035	0,0016	0,0016
BM2	0,2387	0,2778	0,7175	0,0251	0,0107	0,0094

Table 8: Estimates and MSE for a GARCH(1,1) model with additive outliers, $p = 5\%$ and outlier size $d = 7$.

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	1,0108	0,1636	0,7752	2,0541	0,0063	0,0177
t-Garch [7]	0,4139	0,0771	0,8226	0,4133	0,0149	0,0186
LAD	0,4299	0,0672	0,7932	0,3918	0,0186	0,0292
SML	0,4042	0,1323	0,7260	0,2772	0,0037	0,0131
M1	0,4678	0,1633	0,7137	0,3394	0,0088	0,0312
M2	0,5337	0,1854	0,6864	0,5371	0,0146	0,0501
BM1	0,1225	0,1849	0,7849	0,0107	0,0022	0,0027
BM2	0,2655	0,2831	0,7149	0,0383	0,0110	0,0096

Table 9: Estimates and MSE for a GARCH(1,1) model with additive outliers, $p = 10\%$ and outlier size $d = 3$.

[6] Estimated degrees of freedom $=2,7978$
[7] Estimated degrees of freedom $=2,8716$

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	2,6798	0,2250	0,6697	21,3229	0,0026	0,0664
t-Garch [8]	3,3431	0,1007	0,4937	23,4980	0,0291	0,2841
LAD	1,2689	0,0284	0,6545	3,3745	0,0301	0,1216
SML	1,5358	0,0989	0,6200	5,6179	0,0111	0,1106
M1	1,3450	0,1848	0,4736	3,0469	0,0284	0,2179
M2	1,2947	0,1553	0,5409	3,1132	0,0433	0,2005
BM1	0,1011	0,1681	0,7797	0,0049	0,0022	0,0029
BM2	0,2013	0,2794	0,7135	0,0165	0,0115	0,0105

Table 10: Estimates and MSE for a GARCH(1,1) model with additive outliers, $p = 10\%$ and outlier size $d = 5$.

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	3,0073	0,2220	0,7097	45,4205	0,0025	0,0444
t-Garch [9]	2,8432	0,0621	0,6206	20,8822	0,0302	0,2052
LAD	2,1839	0,0140	0,4695	7,5742	0,0329	0,2505
SML	1,8199	0,0338	0,7065	9,6559	0,0289	0,1178
M1	1,7798	0,1359	0,4163	4,8763	0,0430	0,3016
M2	1,6548	0,0858	0,5165	4,8727	0,0489	0,2348
BM1	0,1124	0,1533	0,7820	0,0051	0,0026	0,0024
BM2	0,1817	0,2610	0,7271	0,0132	0,0093	0,0088

Table 11: Estimates and MSE for a GARCH(1,1) model with additive outliers, $p = 5\%$ and outlier size $d = 7$.

[8] Estimated degrees of freedom $= 2,6530$
[9] Estimated degrees of freedom $= 2,8352$

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	1,9785	0,3076	0,6262	9,0437	0,0436	0,0876
t-Garch [10]	1,0365	0,8210	0,2807	1,5402	0,4200	0,2983
LAD	1,4953	1,2343	0,0673	2,2075	1,1110	0,5411
SML	0,4725	0,1492	0,7141	0,4355	0,0038	0,0151
M1	1,1424	0,6745	0,2354	1,3266	0,2408	0,3315
M2	1,0291	0,6767	0,2818	1,0997	0,2477	0,2877
BM1	0,7130	0,5768	0,4205	0,5953	0,1702	0,1657
BM2	0,6025	0,3636	0,6363	0,5279	0,0490	0,0456

Table 12: Estimates and MSE for a GARCH(1,1) model with clustered additive outliers, $p = 5\%$, cluster size $\lambda = 5$ and outlier size $d = 5$.

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	1,7820	0,3052	0,6483	7,2047	0,0415	0,0699
t-Garch [11]	0,7109	0,7982	0,3444	0,7684	0,3934	0,2330
LAD	1,3824	1,3619	0,0795	1,8965	1,3930	0,5236
SML	0,5210	0,1504	0,7117	0,5744	0,0041	0,0170
M1	1,0118	0,6754	0,2647	0,9968	0,2422	0,2978
M2	0,8741	0,6590	0,3205	0,7639	0,2317	0,2463
BM1	0,7759	0,5974	0,4020	0,6464	0,1884	0,1810
BM2	0,6577	0,3530	0,6467	0,6973	0,0449	0,0420

Table 13: Estimates and MSE for a GARCH(1,1) model with clustered additive outliers, $p = 5\%$, cluster size $\lambda = 7$ and outlier size $d = 5$.

[10] Estimated degrees of freedom $=4,2231$
[11] Estimated degrees of freedom $=4,5072$

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	2,9273	0,2398	0,6732	25,7523	0,0079	0,0506
t-Garch [12]	0,5142	0,6745	0,4766	1,1128	0,2582	0,1306
LAD	2,1560	1,3550	0,0147	4,6726	1,3687	0,6172
SML	0,6895	0,1481	0,7160	1,5447	0,0052	0,0197
M1	1,5101	0,7598	0,1498	2,4057	0,3313	0,4359
M2	1,2595	0,7634	0,1972	1,6929	0,3420	0,3849
BM1	0,8068	0,4860	0,5133	0,9048	0,1183	0,1133
BM2	0,8008	0,3438	0,6559	1,3830	0,0438	0,0410

Table 14: Estimates and MSE for a GARCH(1,1) model with clustered additive outliers, $p = 10\%$, cluster size $\lambda = 3$ and outlier size $d = 5$.

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	2,5030	0,2396	0,6990	18,1260	0,0094	0,0342
t-Garch [13]	0,1482	0,5428	0,5952	0,1936	0,1357	0,0496
LAD	2,0168	1,5813	0,0189	4,0546	1,9448	0,6111
SML	0,6963	0,1501	0,7123	2,2460	0,0045	0,0186
M1	1,2312	0,7614	0,2031	1,5852	0,3353	0,3695
M2	0,9745	0,7316	0,2599	0,9851	0,3076	0,3084
BM1	0,9474	0,4455	0,5539	1,6497	0,0973	0,0929
BM2	0,8789	0,3191	0,6805	1,9667	0,0307	0,0284

Table 15: Estimates and MSE for a GARCH(1,1) model with clustered additive outliers, $p = 10\%$, cluster size $\lambda = 5$ and outlier size $d = 5$.

[12] Estimated degrees of freedom $=4,7857$
[13] Estimated degrees of freedom $=4,9731$

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	2,0245	0,2396	0,7222	13,5743	0,0102	0,0249
t-Garch [14]	0,1278	0,4989	0,6276	0,2647	0,1067	0,0377
LAD	1,9778	1,6690	0,0291	4,1736	2,1961	0,5959
SML	0,6442	0,1507	0,7111	1,6802	0,0040	0,0168
M1	0,9580	0,7127	0,2728	0,9480	0,2843	0,2916
M2	0,7321	0,6583	0,3399	0,5539	0,2325	0,2254
BM1	1,0324	0,4278	0,5712	1,9431	0,0901	0,0865
BM2	0,9112	0,3114	0,6881	2,2586	0,0265	0,0245

Table 16: Estimates and MSE for a GARCH(1,1) model with clustered additive outliers, $p = 5\%$, cluster size $\lambda = 7$ and outlier size $d = 5$.

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	NA	NA	NA	NA	NA	NA
t-Garch [15]	0,1270	0,1948	0,7955	0,0039	0,0009	0,0007
LAD	0,1324	0,1885	0,7924	0,0074	0,0021	0,0020
SML	0,2642	0,1411	0,7297	0,0390	0,0024	0,0050
M1	0,1423	0,2037	0,7819	0,0047	0,0020	0,0017
M2	0,2312	0,2446	0,7507	0,0246	0,0052	0,0042
BM1	0,1770	0,2407	0,7534	0,0102	0,0043	0,0035
BM2	0,3607	0,2785	0,7213	0,0736	0,0088	0,0072

Table 17: Estimates and MSE for a GARCH(1,1) model with outliers in the innovations, $p = 5\%$ and $n = 3$.

[14] Estimated degrees of freedom $=4,9806$
[15] Estimated degrees of freedom =150

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	NA	NA	NA	NA	NA	NA
t-Garch [16]	0,1236	0,1945	0,7948	0,0035	0,0010	0,0009
LAD	0,1209	0,1920	0,7930	0,0057	0,0023	0,0021
SML	0,2594	0,1405	0,7307	0,0337	0,0025	0,0049
M1	0,1371	0,2058	0,7805	0,0041	0,0020	0,0018
M2	0,2225	0,2441	0,7519	0,0221	0,0052	0,0043
BM1	0,1659	0,2391	0,7551	0,0074	0,0042	0,0035
BM2	0,3433	0,2789	0,7209	0,0650	0,0090	0,0074

Table 18: Estimates and MSE for a GARCH(1,1) model with outliers in the innovations, $p = 5\%$ and $n = 4$.

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	NA	NA	NA	NA	NA	NA
t-Garch [17]	0,1360	0,2009	0,7952	0,0055	0,0012	0,0007
LAD	0,1423	0,1961	0,7908	0,0167	0,0024	0,0018
SML	0,2563	0,1427	0,7266	0,0419	0,0023	0,0055
M1	0,1618	0,2087	0,7803	0,0109	0,0022	0,0017
M2	0,2631	0,2452	0,7507	0,0393	0,0051	0,0041
BM1	0,2246	0,2513	0,7461	0,0244	0,0048	0,0037
BM2	0,3884	0,2682	0,7317	0,0881	0,0069	0,0054

Table 19: Estimates and MSE for a GARCH(1,1) model with outliers in the innovations, $p = 10\%$ and $n = 3$.

[16] Estimated degrees of freedom =150
[17] Estimated degrees of freedom =150

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	NA	NA	NA	NA	NA	NA
t-Garch [18]	0,1337	0,1997	0,7937	0,0048	0,0011	0,0008
LAD	0,1392	0,1936	0,7890	0,0094	0,0024	0,0023
SML	0,2693	0,1422	0,7275	0,0492	0,0023	0,0054
M1	0,1549	0,2105	0,7771	0,0080	0,0023	0,0019
M2	0,2557	0,2480	0,7478	0,0348	0,0055	0,0044
BM1	0,1951	0,2457	0,7498	0,0150	0,0045	0,0036
BM2	0,3781	0,2724	0,7273	0,0829	0,0078	0,0063

Table 20: Estimates and MSE for a GARCH(1,1) model with outliers in the innovations, $p = 10\%$ and $n = 4$.

Estimator	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	MSE(α_0)	MSE(α_1)	MSE(β_1)
QML	0,1543	0,2056	0,7859	0,0064	0,0013	0,0011
t-Garch [19]	0,1274	0,1957	0,7957	0,0029	0,0010	0,0008
LAD	0,1278	0,1864	0,7955	0,0078	0,0023	0,0025
SML	0,2523	0,1410	0,7297	0,0301	0,0024	0,0051
M1	0,1461	0,2063	0,7803	0,0059	0,0021	0,0019
M2	0,2397	0,2443	0,7513	0,0283	0,0051	0,0042
BM1	0,1768	0,2377	0,7564	0,0098	0,0038	0,0032
BM2	0,3656	0,2777	0,7222	0,0761	0,0087	0,0071

Table 21: Estimates and MSE for a GARCH(1,1) model with outliers in the innovations, $p = 10\%$ and $n = 5$.

[18] Estimated degrees of freedom =150
[19] Estimated degrees of freedom =150

Results of the empirical study

Stock	KS	JB	LM	RLM	BP	LB
	p-value					
Agilent Technologies Inc.	0.0000	0.0000	$0,0002$	$0,0452$	$0,0004$	$0,0004$
Integralis AG	0.0000	0.0000	0.0000	$0,0003$	0.0510	0.0509
Bobst Group AG	0.0000	0.0000	0.0000	$0,6191$	0.0000	0.0000
Cerespo Co., Ltd.	0.0000	0.0000	0.0000	$0,0539$	0.0000	0.0000

Table 22: P-value of different tests.

Stock	Method	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$
Agilent Technologies Inc.				
	QML	0,00000945	0,0964	0,9036
	LAD	0,00001802	0,1008	0,6288
	SML	0,00019599	0,5012	0,4988
	M1	0,00000265	0,0357	0,9470
	M2	0,00000011	0,0380	0,9533
	BM1	0,00000245	0,0622	0,9315
	BM2	0,00001433	0,1756	0,8244
Integralis AG				
	QML	0,0001927	0,3425	0,6575
	LAD	0,0000006	0,0502	0,8566
	SML	0,0009554	0,6396	0,3604
	M1	0,0000114	0,0763	0,9053
	M2	0,0000049	0,1134	0,8585
	BM1	0,0000106	0,1259	0,8741
	BM2	0,0000200	0,1552	0,8448
Bobst Group AG				
	QML	0,0000416	0,1475	0,7379
	LAD	0,0000748	0,0905	0,0011
	SML	0,0000678	0,2393	0,7124
	M1	0,0000356	0,1315	0,6860
	M2	0,0000653	0,1665	0,5373
	BM1	0,0000389	0,1938	0,6352
	BM2	0,0000537	0,2594	0,5593
Cerespo Co. Ltd.				
	QML	0,000221	0,3905	0,6095
	LAD	0,000036	0,1015	0,6873
	SML	0,000833	0,6301	0,3699
	M1	0,000049	0,2078	0,7364
	M2	0,000052	0,2818	0,6905
	BM1	0,000037	0,2484	0,7445
	BM2	0,000034	0,1778	0,8222

Table 23: GARCH parameter for different stocks.

Stock	Method	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\beta}_1$	
Agilent Technologies Inc.					
	QML	0,0003	0,7303	0,0204	0,0140
	LAD	-0,0052	2,9005	0,0231	0,0153
	SML	-0,0050	0,6141	0,0281	0,0186
	M1	-0,0007	1,0403	0,0180	0,0127
	M2	0,0015	0,9747	0,0153	0,0111
	BM1	0,0041	1,0420	0,0186	0,0132
	BM2	0,0018	1,0605	0,0198	0,0134
Integralis AG					
	QML	-0,0253	0,7803	0,0053	0,0035
	LAD	-0,0406	2,8476	-0,0049	-0,0037
	SML	-0,0256	0,5832	0,0056	0,0037
	M1	-0,0236	1,0442	-0,0086	-0,0065
	M2	-0,0260	1,2041	-0,0047	-0,0036
	BM1	-0,0264	1,0538	-0,0086	-0,0064
	BM2	-0,0307	1,2476	-0,0062	-0,0048
Bobst Group AG					
	QML	0,0225	0,7983	-0,1379	-0,0928
	LAD	0,0361	2,6610	-0,1311	-0,0872
	SML	0,0185	0,5427	-0,1376	-0,0923
	M1	0,0262	1,1116	-0,1370	-0,0917
	M2	0,0246	1,0666	-0,1349	-0,0900
	BM1	0,0273	1,1280	-0,1345	-0,0896
	BM2	0,0267	1,0780	-0,1328	-0,0887
Cerespo Co., Ltd.					
	QML	-0,008	0,7681	-0,0825	-0,0554
	LAD	-0,015	2,5970	-0,0840	-0,0566
	SML	-0,010	0,5845	-0,0804	-0,0540
	M1	-0,009	1,1366	-0,0839	-0,0566
	M2	-0,007	1,0611	-0,0810	-0,0545
	BM1	-0,010	1,1192	-0,0820	-0,0556
	BM2	-0,015	1,1482	-0,0835	-0,0574

Table 24: mean, variance, spearmans ρ and kenndals τ for different stocks.

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